# Random Beamforming Combined with Receive Beamforming in mmWave Multiuser MIMO Downlink

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***Abstract*—The performance of transmit random beamforming (RBF) with receive beamforming at each user is investigated for millimeter wave (mmWave) multiuser multiple-input multiple-output (MU-MIMO) downlink systems. It is shown that RBF in MU-MIMO systems with the total transmit power inversely proportional to the number of receive antennas can achieve the performance of the conventional RBF scheme with the constant total transmit power in MU multiple-input single-output (MISO) systems with no receive beamforming, when the number of transmit antennas is large.**

***Keywords*—millimeter wave systems, multiuser MIMO downlink, random beamforming, sparse MIMO channels**

## I. Introduction

The use of mmWave bands is considered as a key technology for the fifth generation (5G) communication systems due to the large bandwidth available at the mmWave frequency band. MmWave signals experience severe path loss and have sparse scattering conditions as compared to signals at the conventional lower cellular frequency bands [1]. To obtain higher data rates in mmWave systems, directional beamforming using large-scale MIMO is crucial to overcome the large path loss in sparse mmWave channels [2]–[4]. However, accurate channel estimation and large channel state information (CSI) feedback are required for downlink beamforming, and result in heavy system overhead.

One way to reduce this feedback burden and circumvent accurate channel estimation in MU-MISO downlink is to exploit multiuser (MU) diversity with partial CSI feedback from each user [5]–[9]. Recently, Lee *et al.* showed that significant MU diversity gains can be obtained in sparse mmWave channels even in large-scale MIMO systems [10], [11]. When the channel vector between the BS and each user is modeled as the uniform-random multi-path (UR-MP) channel model that captures the property of sparse mmWave channels, random beamforming (RBF) can achieve linear sum-rate scaling with respect to (w.r.t.) the number $M$ of transmit antennas under the MISO situation if the number $K$ of users in the cell *linearly* increases w.r.t. $M$ as $M \to \infty$. This result is contrary to the existing result that linear sum-rate scaling w.r.t. $M$ can be achieved only if the number $K$ of users increases *exponentially* w.r.t. $M$ under the i.i.d. Rayleigh fading channel model capturing rich-scattering environments [5], and sheds a positive prospect on harnessing the MU diversity gain with partial CSI in sparse mmWave large-scale MIMO channels.

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However, the previous result on the MU diversity gain in mmWave MU-MISO downlink systems is limited to the case in which each user has a single receive antenna [10], [11]. In mmWave systems, multiple antennas are likely to be equipped at each user too due to the fact that the shorter wavelength at the mmWave frequency band can enable more antennas to be packed in a smaller form. Thus, receive beamforming from the multiple receive antennas at each user in addition to the transmit beamforming at the base station can be adopted to compensate for the large path loss and suppress the inter-user interference. In this paper, we extend the result in [10], [11] and analyze the performance of RBF in the mmWave MU-MIMO downlink system where the base station uses random transmit beamforming based on the transmit antenna array and each user has multiple received antennas and employs receive beamforming based on the multiple antennas.

## II. System Model and Performance Analysis

We consider a MU-MIMO downlink system where the BS with $M$ transmit antennas is synchronized with $K(\geq M)$ users with $N$ receive antennas each. We assume that the antenna configuration of the BS and each user $k$ is the uniform linear array (ULA), and the channel matrix between the BS and user $k$ is given by

$$\mathbf{H}_k = \sqrt{\frac{MN}{L}} \sum_{i=1}^{L} \alpha_{k,i} \mathbf{a}_R(\theta_{k,i}^r) \mathbf{a}_T(\theta_{k,i}^t)^H. \qquad (1)$$

Here, $L$ is the number of multiple paths, the $i$-th path gain of user $k$ is modeled as independent and identically distributed (i.i.d.) complex Gaussian, i.e., $\alpha_{k,i} \overset{i.i.d.}{\sim} \mathcal{CN}(0,1)$, the normalized angle-of-arrival (AoA) and angle-of-departure (AoD) of path $i$ of user $k$ are generated according to the uniform distribution on the interval $[-1,1)$, i.e., $\theta_{k,i}^r, \theta_{k,i}^t \overset{i.i.d.}{\sim} \text{Unif}[-1,1]$, respectively, and the array steering vectors are given by

$$\mathbf{a}_R(\theta) = \frac{1}{\sqrt{N}}[1, e^{-\iota\pi\theta}, \cdots, e^{-\iota\pi(N-1)\theta}]^T$$
$$\mathbf{a}_T(\theta) = \frac{1}{\sqrt{M}}[1, e^{-\iota\pi\theta}, \cdots, e^{-\iota\pi(M-1)\theta}]^T, \quad \iota = \sqrt{-1}$$

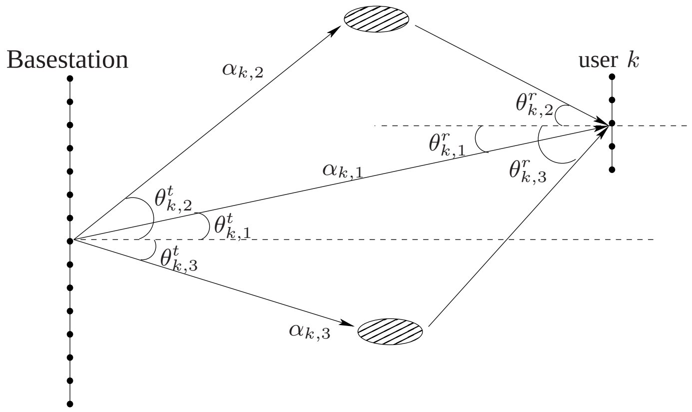


Fig. 1. The considered channel model when $L = 3$.

where the normalized AoA (AoD) $\theta \in [-1, 1]$ is related to the physical AoA (AoD) $\phi \in [-\pi/2, \pi/2]$ as [12]

$$\theta = \frac{2d\sin(\phi)}{\lambda} \tag{2}$$

and $d$ and $\lambda$ are the distance between two adjacent antenna elements and the carrier wavelength, respectively, with the critical spatial sampling of $d/\lambda = 1/2$ assumed in this paper. The considered channel model is illustrated in Fig. 1.

In this paper, we extend and modify RBF used in MU-MISO to the MU-MIMO case, where each user has multiple antennas and uses receive beamforming, as follows.

S.1 The BS generates a set of orthogonal beam vectors and transmits each element of the set sequentially during the training period. We assume each beam vector $\mathbf{w}_b$ is designed as

$$\mathbf{u}_b = \mathbf{a}_T(\vartheta_b) = \mathbf{a}_T\left(\vartheta + \frac{2(b-1)}{M}\right), \tag{3}$$

for $b = 1, \cdots, M$, where $\vartheta \sim \text{Unif}[-1, 1]$ or equivalently $\vartheta \sim \text{Unif}[-1, -1 + \frac{2}{M}]$ is a random offset value.

S.2 After the training is over, based on $\{\mathbf{H}_k\mathbf{u}_b\}_{b=1}^{M}$ each user $k$ computes

$$\{\mathbf{v}_k, b_k\} = \arg\max_{\mathbf{v},b} \text{SINR}_b(\mathbf{v}), \tag{4}$$

where $\text{SINR}_b(\mathbf{v})$ is the signal-to-interference-plus-noise ratio (SINR) value with a unit-norm receive beamforming vector $\mathbf{v}$ for transmit beam $b$, i.e.,

$$\text{SINR}_b(\mathbf{v}) = \frac{\rho|\mathbf{v}^H\mathbf{H}_k\mathbf{u}_b|^2}{1 + \rho\sum_{b'\neq b}|\mathbf{v}^H\mathbf{H}_k\mathbf{u}_{b'}|^2}. \tag{5}$$

Here, we assume that the total transmit power $P_t$ is equally allocated to each data stream, i.e., $\rho = \frac{P_t}{M}$. Then, the optimal solution $\mathbf{v}_k$ is given by [13]

$$\mathbf{v}_k = \eta\left(\rho\sum_{b'\neq b_k}\mathbf{H}_k\mathbf{u}_{b_k}\mathbf{u}_{b_k}^H\mathbf{H}_k^H + \mathbf{I}\right)^{-1}\mathbf{H}_k\mathbf{u}_{b_k}, \tag{6}$$

where $\eta$ is the power scaling factor to satisfy $\|\mathbf{v}_k\| = 1$. Each user $k$ feeds the maximum SINR value, i.e., $\text{SINR}_{b_k}(\mathbf{v}_k)$ and its corresponding transmit beam index $b_k$ back to the BS, and stores its receive beamforming vector $\mathbf{v}_k$.

S.3 After the feedback from each user is over, the BS chooses the user that has the maximum SINR value among the users feeding its beam index as $b$ for each transmit beam $b$ and transmits data streams to the chosen users with the transmit beam vectors $\{\mathbf{u}_b\}$. At each selected user $k$, it receives a data stream by using its stored receive beamforming vector $\mathbf{v}_k$.

Note that in the proposed scheme, $N$ RF chains are assumed to be available so that the optimal receive beamformer can be obtained from $\mathbf{H}_k\mathbf{u}_b$. Note also that the information about optimal receive beamformer need not be fed back to the base station. In the considered RBF scheme for the mmWave MU-MIMO downlink system, the expected sum rate is given by

$$\mathcal{R}_{sum} = \sum_{b=1}^{M}\mathcal{R}_{\kappa_b} = \sum_{b=1}^{M}\mathbb{E}\left[\log\left(1 + \text{SINR}_b(\mathbf{v}_{\kappa_b})\right)\right] \tag{7}$$

where $\kappa_b = \arg\max_{\{k:b=b_k\}}\text{SINR}_b(\mathbf{v}_k)$.

To model the large-scale antenna array at the BS for high-gain beamforming at the mmWave band, we consider the asymptotic scenario in which the number $M$ of transmit antennas at the BS tends to infinity with $L$ and $N$ as functions of $M$. The following theorem shows the impact of receive beamforming on RBF under this asymptotic scenario.

*Theorem 1:* Under the considered sparse mmWave MU-MIMO channel model (1), if the following conditions are satisfied:

(C.1) $L$ tends to infinity as $M \to \infty$,

(C.2) $\frac{L}{M} \leq 1$,

(C.3) $K = Me^{cL}$, for some $c > 3$,

then an asymptotic lower bound on $\mathcal{R}_{\kappa_b}$ for $P_t = \frac{1}{N}$ is given by

$$\mathcal{R}_{\kappa_b} \gtrsim r_{LB} > 0, \tag{8}$$

where $x \gtrsim y$ indicates $\lim_{M\to\infty} x/y \geq 1$ and $r_{LB}$ is a positive constant value.

*Proof:* * Let $A$ be the event $\bar{A}$ defined in Appendix D in [11]. Then, the asymptotic probability of the event $A$ is given by [11]

$$\Pr\{A\} \to 1, \text{ as } M \to \infty. \tag{9}$$

From the fact that $\mathbb{E}[f(X)] \geq p(A)\mathbb{E}[f(X|A)]$ for a non-negative function $f(X)$, $\mathcal{R}_{\kappa_b}$ is lower bounded by

$$\mathcal{R}_{\kappa_b} \geq \Pr\{A\}\mathbb{E}\left[\log\left(1 + \text{SINR}_b(\mathbf{v}_{\kappa_b})\right)\Big|A\right]. \tag{10}$$

*The overall procedure of this proof is similar to the proof of Theorem 3 in [11] but additional bounding techniques for SINR with the receive beamforming and beam gains are newly applied to this proof. Hence, in this proof, we borrowed the overall steps of the proof of Theorem 3 in [11].

Furthermore, the second term in the right-hand side (RHS) of (10) is bounded as

$$\begin{aligned}
&\mathbb{E}\left[\log\left(1+\mathrm{SINR}_b(\mathbf{v}_{\kappa_b})\right)\Big|A\right] \\
&\overset{(a)}{\geq} \mathbb{E}\left[\log\left(1+\mathrm{SINR}_b(\mathbf{v}_k)\right)\Big|A\right] \\
&\overset{(b)}{\geq} \mathbb{E}\left[\log\left(1+\mathrm{SINR}_b(\mathbf{a}_R(\theta^r_{k,1}))\right)\Big|A\right], \qquad (11)
\end{aligned}$$

where $(a)$ follows from the fact that the rate of user $k$ for beam $b$ is not larger than that of the optimal user $\kappa_b$ for beam $b$, and $(b)$ is due to $\mathrm{SINR}_b(\mathbf{v}_k) = \max_{\mathbf{v}} \mathrm{SINR}_b(\mathbf{v}) \geq \mathrm{SINR}_b(\mathbf{a}_R(\theta^r_{k,1}))$. Since $\Pr\{A\} \to 1$ as $M \to \infty$, it is now left to show

$$\mathbb{E}\left[\log\left(1+\frac{X}{1+Y}\right)\Big|A\right] \gtrsim r_{LB} \text{ as } M \to \infty, \qquad (12)$$

where

$$X = \frac{1}{L}\left|\sum_{i=1}^{L} \alpha_{k,i} G_R(\theta^r_{k,1}, \theta^r_{k,i}) G_T(\theta^t_{k,i}, \vartheta_b)\right|^2 \qquad (13)$$

and

$$Y = \frac{1}{L}\sum_{b' \neq b}\left|\sum_{i=1}^{L} \alpha_{k,i} G_R(\theta^r_{k,1}, \theta^r_{k,i}) G_T(\theta^t_{k,i}, \vartheta_{b'})\right|^2, \qquad (14)$$

and $G_R(\theta_1, \theta_2) = \mathbf{a}_R^H(\theta_1)\mathbf{a}_R(\theta_2)$ and $G_T(\theta_1, \theta_2) = \mathbf{a}_T^H(\theta_1)\mathbf{a}_T(\theta_2)$. From the fact that $G_R(\theta_1, \theta_2) \leq 1$ and the equality holds when $\theta_1 = \theta_2$, we directly find an asymptotic lower bound on $X$ as

$$X \gtrsim \frac{4}{\pi^2} \qquad (15)$$

by using the same techniques used in (56)-(58) in [11]. Also, we can compute an asymptotic upper bound on $\mathbb{E}[Y|A]$ as

$$\mathbb{E}[Y|A] \lesssim \frac{4\pi^2}{3} \qquad (16)$$

using the same techniques used in (60)-(62) in [11], where $x \lesssim y$ indicates $\lim_{M\to\infty} x/y \leq 1$. Therefore, we have

$$\begin{aligned}
\mathbb{E}\left[\log\left(1+\frac{X}{1+Y}\right)\Big|A\right] &\overset{(a)}{\gtrsim} \mathbb{E}\left[\log\left(1+\frac{4/\pi^2}{1+Y}\right)\Big|A\right] \\
&\overset{(b)}{>} \log\left(1+\frac{4/\pi^2}{1+\mathbb{E}[Y|A]}\right) \\
&\overset{(c)}{\gtrsim} \log\left(1+\frac{4/\pi^2}{1+4\pi^2/3}\right) =: r_{LB} \qquad (17)
\end{aligned}$$

where $(a)$ follows from (15), $(b)$ is by the convexity of the function $f(x) = \log(1+\frac{1}{1+x})$ and Jensen's inequality, and $(c)$ is due to (16). This concludes the proof. ■

Theorem 1 states that under the considered sparse mmWave MU-MIMO channel model for multiple-antenna receivers, the performance of the considered RBF scheme with receive beamforming and the total transmit power $P_t = \frac{1}{N}$ has the same behavior as that of RBF with $P_t = 1$ in the MU-MISO system under the UR-MP channel model [11]. Theorem 1 also specifies the sufficient number $K$ of users to achieve linear sum rate scaling by the considered RBF scheme w.r.t. $M$ for different levels of sparsity $L$ in mmWave MU-MIMO channels. Since the conditions (C.1), (C.2) and (C.3) are the same as those required for Theorem 3 of [11], the sufficient number of users required for linear sum rate scaling w.r.t. $M$ for the MU-MIMO case is the same as that required for the MU-MISO case,[†] but the required transmit power is reduced by factor $N$, i.e., the number of receive antennas.

## III. Conclusions

In this paper, we have considered mmWave MU-MIMO downlink where both the base station and users have multiple antennas. We have extended the conventional RBF scheme considered in the MU-MISO case to the MU-MIMO case with optimal receive beamforming at each user and have analyzed the performance of the extended RBF scheme. We have shown that the MU-MIMO RBF scheme with the total transmit power inversely proportional to the number $N$ of receive antennas has the same performance as the conventional MU-MISO RBF scheme with constant total transmit power, and employing receive beamforming is beneficial to reduce the total transmit power at the base station in mmWave MIMO systems.

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[†]Please see Table 1 in [11].